Balancing of a rigid rotor using artificial neural network to predict the correction masses

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ABSTRACT. This paper deals with an analytical model of a rigid rotor supported by hydrodynamic journal bearings where the plane separation technique together with the Artificial Neural Network (ANN) is used to predict the location and magnitude of the correction masses for balancing the rotor bearing system. The rotating system is modeled by applying the rigid shaft Stodola-Green model, in which the shaft gyroscopic moments and rotatory inertia are accounted for, in conjunction with the hydrodynamic cylindrical journal bearing model based on the classical Reynolds equation. A linearized perturbation procedure is employed to render the lubrication equations from the Reynolds equation, which allows predicting the eight linear force coefficients associated with the bearing direct and cross-coupled stiffness and damping coefficients. The results show that the methodology presented is efficient for balancing rotor systems. This paper gives a step further in the monitoring process, since Artificial Neural Network is normally used to predict, not to correct the mass unbalance. The procedure presented can be used in turbo machinery industry to balance rotating machinery that require continuous inspections. Some simulated results will be used in order to clarify the methodology presented.

Key words: rigid balancing, rotor balancing, artificial neural network.

Introduction

Rotor unbalance is a common source of vibration in turbomachinery. The rotor mass unbalance transmits rotating forces to the bearings and foundations. Such forces may damage the system and in some cases even affect other equipments in the vicinity. Rigid and flexible rotors can be balanced by specific balancing techniques. Plane separation technique is normally used for balancing rigid rotors, while Modal Balancing Method and Influence Coefficient Method can be used for balancing flexible rotors (RAO, 1983; VANCE, 1988; CHILDS, 1993). Parkinson (1991) describes the unbalance vibration process, in addition to discussing about balancing techniques for rigid and flexible rotors.

Artificial Intelligence (AI), especially Artificial Neural Network (ANN) techniques, has been used for monitoring and fault diagnostics of mechanical systems (PANTELELIS et al., 2000). For that, experimental or theoretical data can be used. Ganesan et al. (1995) applied ANN on diagnostics and instability control of
high-speed rotating systems using analytical model with great success. The results presented in Vyas and Satishkumar (2001) also showed the great capability of the artificial neural network in fault prediction found in rotating machines. Paya and Esat (1997) applied wavelet transform to preprocess six different types of vibration signals obtained from a model drive line, consisting of various interconnected rotating parts; from the preprocessed data, an ANN was used to determine the health condition of the system. The results showed that ANN using preprocessed data by wavelet transform was successfully in detection and classification of single and multiple faults. Samanta and Al-Balushi (2003) proposed a procedure for fault diagnosis of rolling element bearings through ANN. Time-domain vibration signals of the rotating machinery, considering the normal and defective bearing condition, were used as input of the ANN. The results presented the effectiveness of the ANN in condition monitoring and diagnostics of machines. In spite of the research published on monitoring and fault detection using artificial intelligence techniques, there are few studies in which such techniques are used to predict the correction masses for balancing mechanical systems.

This paper deals with the balancing of a rigid rotor supported by hydrodynamic journal bearings by using plane separation technique and artificial neural networks. The rotating shaft model is based on the Stodola-Green model, which takes into account the shaft gyroscopic moments and rotational inertia. The bearing dynamic model is based on the perturbed lubrication equations derived from the classical Reynolds equation (LUND, 1987). The bearing model permits to predict eight bearing force coefficients, four stiffness and four damping coefficients, associated with the lateral rotor motion, including the cross-coupled dynamic force coefficients. An artificial neural network is trained using the unbalance responses of the rotor bearing system together with the correction masses provided by the plane separation balancing technique. After that, the artificial neural network is able to predict the correction masses when the unbalance responses are provided to neural network.

**Material and methods**

**Model**

The equations of the lateral motion for a rigid rotor can be obtained from the Lagrangian determined in terms of the Euler angles, as presented in Figure 1. Initially the body-fixed reference frame $xyz$ and the space-fixed inertial frame $XYZ$ are coincident. The order of the rotations for the Lagrangian is the following Vance (1988): a) $\alpha$ rotation about $y$; b) $\beta$ rotation about $x$; c) $\psi$ rotation about $z$.

![The Euler angles.](image)

The four degrees-of-freedom accounted for in the rigid rotor model describe the bending vibration. The Lagrangian composed by the translational and rotational kinetic energy for the rotor bearing system is given by Equation 1:

$$L = T = \frac{1}{2} M (x^2 + y^2) + \frac{1}{2} I_t (\alpha^2 + \beta^2) + \frac{1}{2} I_p (\alpha^2 + 2\omega \alpha \beta)$$

(1)

where:

$L$ is the shaft Lagrangian;
$T$ represents the rotor kinetic energy;
$M$ is the rotor mass;
$I_t$ is the transverse mass moment of inertia;
$I_p$ is the polar mass moment of inertia.

The shaft rotating speed is given by $\omega$.

In this work, the rotor bearing system model considers the Stodola-Green shaft model in which a rigid rotor is supported by hydrodynamic journal bearings localized on $Z = \pm L/2$, according to Figure 2.

![Rigid rotor.](image)

The equations of motion (Equation (2)) for the rotor bearing system are obtained using a Lagrangian formulation (VANCE, 1988), with $X, Y, \alpha e \beta$ as the generalized coordinates of the system and $F$ and $M$ as the generalized forces and moments, respectively.
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The terms involving the polar mass moment of inertia ($I_p$) are known as gyroscopic moments.

\[
M\ddot{X} = \sum F_x
\]
\[
M\ddot{Y} = \sum F_y
\]
\[
I_\beta + I_\alpha \omega \dot{\beta} = \sum M_x
\]
\[
I_\alpha - I_\beta \omega \dot{\alpha} = \sum M_y
\]

The right-hand side (RHS) terms of Equation (2) represent the generalized forces and momentum acting on the rotor, which are due to the bearing reaction forces and to the centrifugal forces associated with the mass unbalance.

The bearing reaction forces are computed from the classical Reynolds equation. A linearized perturbation procedure is applied on the Reynolds equation in order to render the zeroth- and first-order lubrication equations, which are represented by Equations (3) and (4), respectively. These equations allow predicting the bearing dynamic force coefficients associated with the rotor lateral motions. A finite element procedure specially devised to compute the bearing dynamic force coefficients is used to render the eight stiffness and damping coefficients (FARIA, 2001).

\[
\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \rho h \frac{\partial h}{\partial \theta} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \rho h \frac{\partial h}{\partial \theta} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \rho h \frac{\partial h}{\partial \theta} \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \rho h \frac{\partial h}{\partial \theta} \right) = \frac{1}{\alpha} \frac{\partial}{\partial \theta} (\rho \theta) \tag{3}
\]

\[
\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} (\rho h \frac{\partial h}{\partial \theta}) \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} (\rho h \frac{\partial h}{\partial \theta}) \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} (\rho h \frac{\partial h}{\partial \theta}) \right) + \frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{\partial}{\partial \theta} (\rho h \frac{\partial h}{\partial \theta}) \right) = \frac{1}{\alpha} \frac{\partial}{\partial \theta} (\rho \theta) + \frac{1}{\alpha} \frac{\partial}{\partial \theta} (i \omega \rho \theta) \tag{4}
\]

The bearing dynamic reaction forces are calculated considering the velocity and displacement in the X and Y direction, respectively. The cross-coupled stiffness (damping) coefficients are based on the fact that a displacement (velocity) in the X direction produces a force in the Y direction, and vice-versa. It is known (VANCE, 1988) that the cross-coupled stiffness coefficients have opposite sign ($K_{xy} = -K_{yx}$ at concentric position), whereas the cross-coupled damping coefficients have the same sign ($C_{xy} = C_{yx}$ at concentric position). However, it is important to point out here that the coupled coefficients are normally ignored in general modeling formulations, although they have a strong influence in the rotor response. So, Equation (5) represents the generalized forces and moments due to the bearing damping and stiffness coefficients, including the coupled coefficients.

\[
\sum F_x = -2C_{xy} \ddot{x} - 2C_{xy} \ddot{y} - 2K_{xy} x - 2K_{xy} y \tag{5}
\]

Therefore, the equations of motion for the system (Equation (6)) can be obtained substituting Equation (5) in (2) and including the generalized forces and moments caused by the rotor unbalance. The latter is represented by the RHS of Equation (6).

\[
M\ddot{X} + 2C_{xy} \ddot{Y} + 2C_{xy} X + 2K_{xy} Y = \sum m_i (\omega^2 \cos(\alpha + \psi) \tag{6}
\]

In the right-hand side of Equation (6), $m_i$ represents the discrete unbalance mass, $u_i$ represents the unbalance eccentricity and $\psi$ represents the phase angle of the unbalance mass. The rotor element length is given by $l$.

The bearing cross-coupled stiffness coefficients tend to reduce the system effective damping (VANCE, 1988). The larger is the cross-coupled stiffness, the smaller is the external damping acting on the rotor. Consequently, the rotor whirl amplitude tends to become very large when this reduction occurs. That stresses the importance of including all bearing force coefficients in the rotor modeling in order to avoid large deviations on the prediction of the unbalance response.

Integration of the equations of motion

The fourth order Runge-Kutta integration method, as presented by Equation (8), can be used to obtain the time responses of the rotor bearing system, in terms of coordinates $X$, $Y$, $\alpha$ and $\beta$. The following expression applies to coordinate $Y$, but can be extended to the other coordinates straightforward.

\[
y_{i+1} = y_i + \frac{1}{6} (k_{i+1} + 2k_{i+2} + 2k_{i+3} + k_{i+4}) \tag{7}
\]

where:

\[
k_i = f(t_i, y_i) \Delta t
\]
\[
k_i = f(t_i + \frac{1}{2} \Delta t, y_i + \frac{1}{2} k_i \Delta t)
\]
\[
k_i = f(t_i + \frac{3}{2} \Delta t, y_i + \frac{3}{4} k_i \Delta t)
\]
\[
k_i = f(t_i + 2 \Delta t, y_i + k_i \Delta t)
\]

The smaller the time increment considered (Δt), the better the resolution obtained for the time responses. From that, a Fast Fourier Transform (FFT) is used to give the responses of the system in the frequency domain.

For the present study, the responses of the system are obtained for the center of the rotor and for the two journal bearings of the system.

**Balancing equations**

A rigid rotor can be balanced by adding correction masses in any two balancing planes. One technique used for that is the plane separation technique. It is important to emphasize that, for rigid balancing, this technique can be applied only for speeds very below its first critical speed, typically between 100 and 600 rpm (RIEGER, 1988). Figure 3 represents the mass eccentricity distribution along the shaft in which two balancing planes (Pb₁ and Pb₂) are defined.

Equations (9) represent the force and moment equations, considering the balancing planes shown in Figure 3, obtained from the plane separation technique:

\[
\begin{align*}
\hat{R}_1 e^{\phi_1} + \hat{R}_2 e^{\phi_2} + m_r \omega^2 e^{\phi_1} + m_t \omega^2 e^{\phi_2} &= 0 \\
L \hat{R}_1 e^{\phi_1} + (L_s + L_b)m_r \omega^2 e^{\phi_1} + L_t m_r \omega^2 e^{\phi_2} &= 0
\end{align*}
\]  

(9)

\( \hat{R}_1 \) and \( \hat{R}_2 \) represent the bearing reaction forces at the two selected measuring planes and \( \phi_1 \) and \( \phi_2 \) represent their phase angles. The solution of the system of Equations (9) will provide the correction masses \( m_1 \) and \( m_2 \) and their respectively angular positions \( \phi_{1s} \) and \( \phi_{2s} \), which will reduce the rotor unbalance response. The correction mass vectors are represented by the Equations (10)

\[
\begin{align*}
S_1 &= m_1 e^{i \phi_1} \\
S_2 &= m_2 e^{i \phi_2}
\end{align*}
\]  

(10)

**Balancing methodology**

The novelty of the balancing methodology proposed in this work is the application of the artificial neural network to balance rotors. A neural network is a massively parallel, self-adaptive, interconnected network of basic elements called ‘neurons’. Neurons have a simple computational process but the interactions between them allow the ANN to learn from given input sets and their corresponding outputs (HAYKIN, 1999). A Multilayer Perceptron Network, as shown in Figure 4, has greater computational power when compared with one layer Neural Network.

The most commonly used Neural Network training algorithms are the Error Backpropagation and the Levenberg-Marquardt Algorithm. Error Backpropagation training algorithm attempts to minimize the square of the error obtained between the ANN output and the desired output, considering a descent gradient technique to change the synaptic weights.

Two stages are necessary: training and validation. ANN is normally used only to detect the unbalance from the response levels, not to provide the correction masses to balance the rotating shaft.

The training database considered contains the unbalance response obtained by mathematical modeling (Equation (6)) of the rotor bearing system and the correction masses given by any balancing process such as the plane separation technique (Equation (10)). For the validation process, the unbalance responses of the system are used as inputs and the correction masses are the outputs of the artificial neural network.

In order to improve the ANN generalization capability, the input and output data need to be normalized. So, the polar input and output data are transformed to Cartesian data and after that, these Cartesian data are normalized. The normalization is performed according to Equation (11):

\[
N(x) = \frac{N(x) - y_{\text{max}}}{y_{\text{max}} - y_{\text{min}}} \\
N(y) = \frac{N(y) - y_{\text{max}}}{y_{\text{max}} - y_{\text{min}}}
\]  

(11)

where:

\( N(x) \) and \( N(y) \) = \( x \) and \( y \) normalized coordinates;
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\[ R(x) \text{ and } R(y) = x \text{ and } y \text{ coordinates to be normalized; } \]
\[ y_{\text{min}} \text{ and } y_{\text{max}} = \text{maximum and minimum values of the } y \text{ coordinates;} \]
\[ x_{\text{min}} \text{ and } x_{\text{max}} = \text{maximum and minimum values of the } x \text{ coordinates.} \]

**Network design**

There are a lot of possible ANN designs to be considered. Most of the researches dealing with rotor mass balancing include either a single or double hidden layers. For the study presented here, the best ANN design was obtained as described in Table 1. Two hidden layers with 20 neurons were used.

**Table 1. Artificial neural network design.**

<table>
<thead>
<tr>
<th>Layers input-output relationship</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{(\text{bearing 1})})</td>
<td></td>
</tr>
<tr>
<td>(y_{(\text{bearing 1})})</td>
<td></td>
</tr>
<tr>
<td>(x_{(\text{balancing plane 1})})</td>
<td></td>
</tr>
<tr>
<td>(y_{(\text{balancing plane 1})})</td>
<td></td>
</tr>
<tr>
<td>(x_{(\text{bearing 2})})</td>
<td></td>
</tr>
<tr>
<td>(y_{(\text{bearing 2})})</td>
<td></td>
</tr>
<tr>
<td>(x_{(\text{center of the rotor})})</td>
<td></td>
</tr>
<tr>
<td>(y_{(\text{center of the rotor})})</td>
<td></td>
</tr>
</tbody>
</table>

**Artificial neural network parameters**

For the learning procedure the Levenberg-Marquardt algorithm was employed (HAYKIN, 1999). The artificial neural network learning parameters used in this stage are described in Table 2.

**Table 2. Artificial neural network parameters.**

<table>
<thead>
<tr>
<th>Artificial Neural Network Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>transfer function</td>
<td>sigmoid function</td>
</tr>
<tr>
<td>rate of learning</td>
<td>0.05</td>
</tr>
<tr>
<td>increase factor of learning</td>
<td>1.05</td>
</tr>
<tr>
<td>performance goal</td>
<td>0.001</td>
</tr>
<tr>
<td>momentum</td>
<td>0.075</td>
</tr>
<tr>
<td>interactions</td>
<td>3,000</td>
</tr>
</tbody>
</table>

**Results and discussion**

In this study, the rotor bearing system was simulated using theoretical data. The rotor parameters employed in the modeling process are described in Table 3. The length and diameter are such that the rotor can be considered a rigid structure and the material employed simulates a common one found in the industry.

**Table 3. Rotor parameters.**

<table>
<thead>
<tr>
<th>Rotor Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>length (m)</td>
<td>0.6</td>
</tr>
<tr>
<td>diameter (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>mass density(kg m(^{-3}))</td>
<td>7800</td>
</tr>
</tbody>
</table>

The rotor is supported by two identical hydrodynamic journal bearings. The damping and stiffness dynamic coefficients of the bearings employed in system simulation are presented in Table 4. They were found solving the zero and first order Reynolds equations via finite element analysis (LUND, 1987).

**Table 4. Dynamic coefficients.**

<table>
<thead>
<tr>
<th>Journal Bearings Dynamic Coefficients</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness Coefficients (N m(^{-1}))</td>
<td></td>
</tr>
<tr>
<td>(K_{XX})</td>
<td>0.2310E+06</td>
</tr>
<tr>
<td>(K_{XY})</td>
<td>0.2449E+05</td>
</tr>
<tr>
<td>(K_{YY})</td>
<td>0.2020E+06</td>
</tr>
<tr>
<td>(K_{YX})</td>
<td>-0.2182E+05</td>
</tr>
<tr>
<td>Damping Coefficients (N s m(^{-1}))</td>
<td></td>
</tr>
<tr>
<td>(C_{XX})</td>
<td>1.718E+03</td>
</tr>
<tr>
<td>(C_{XY})</td>
<td>1.672E+03</td>
</tr>
<tr>
<td>(C_{YY})</td>
<td>1.713E+03</td>
</tr>
<tr>
<td>(C_{YX})</td>
<td>1.672E+03</td>
</tr>
</tbody>
</table>

From a random mass eccentricity distribution along the shaft, the response of the system is obtained using the parameters given in Table 3 and the dynamic coefficients given in Table 4, considering a balancing speed of 400 rpm. The unbalance responses are obtained at the rotor center and at the bearings. Figure 5 shows the synchronous whirl and frequency spectra for the center of the rotor. The same procedure was done for the bearings position and similar results were obtained.

**Figure 5. The unbalance response.**

The database was generated using the unbalance responses of the system and the correction masses
given by the balancing process, according to the methodology described. From the database, the training and validation sets are obtained.

The training set is composed by 200 elements in which the unbalance responses of the rotor bearing system are the input data and the correction masses are the output data. The convergence of the learning procedure was terminated at 0.1% error threshold. Twenty (20) unknown elements formed the validation set. These elements are the input data and correspond to the unbalancing responses provided to the network. So, the correction masses and angles (output data) are compared with the correct ones to check the quality of the generalization, as shown in Figures 6 and 7.

The results show that the artificial neural networks are able to predict the correction masses with satisfactory accuracy. In general, for the validation set used, the error was below 10% and only for 4 elements of the validation set there was an error above 5%. Nevertheless, the predictions obtained from these 4 elements when applied to the system balancing presented unbalance response reductions up to 74%. So, that confirms the great capability of the methodology presented.

**Conclusion**

A rotor balancing methodology for rigid rotors supported on oil-lubricated journal bearings has been presented. That uses the unbalance response of the system, together with the Artificial Neural Network (ANN) to predict the correction data (masses and angles) to balance the system. That is performed in two stages: training and validation. The methodology proposed gives a step further in the balancing process using the ANN since the later is normally used only to detect the rotor unbalance, not to provide the corrective masses capable of minimizing the rotor vibration response.

The full model of the rotating system together with the correction data obtained from the plane separation balancing technique presented satisfactory results and can be used to constitute the database that is employed to train the ANN. A full model description of the rotating system is important in order to obtain an accurate response prediction of the system, respective to the levels obtained. The rotor model must include the shaft gyroscopic effects and the bearing dynamic force coefficients, mainly the cross-coupled stiffness coefficients, in order to represent more efficiently real rotors supported on hydrodynamic journal bearings.

The better ANN design for the system used was obtained using two hidden layers. The Levenberg-
Marquardt training algorithm presented a good performance within the 0.1% error threshold, showing a satisfactory generalization during the validation stage. Even for generalization results around 10% error there was a reduction of the unbalance responses up to 74%. Therefore, it is possible to conclude about the efficiency of the balancing methodology proposed.

References


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